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Optimization of Geometric Characteristics of Cycloidal Profiles of Gerotor Hydraulic Machines

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Abstract

Introduction. The performance and reliability of gerotor hydraulic machines depend on the geometry of the cycloidal gearing profile. The existing methods of calculating and optimizing the profile parameters are cumbersome, multi-criteria and difficult for practical application. Therefore, the problem of creating the methodology for calculating the parameters of the gerotor machine profile suitable for engineering calculations at the stage of conceptual design is a challenge. In this regard, the objective of this work was to modernize the methodology for designing the geometry of the profiles of the end section of hypocycloidal gears used in gerotor hydraulic machines, and to analyze the possibilities of their optimization during preliminary design. In the course of the study, the Mathcad computer mathematics system was used; numerical experiments were carried out to study the influence of geometric profile parameters on the performance and operability of the gerotor hydraulic machine, based on the data obtained and analyzed; recommendations for the design of optimal profiles of the end section of gerotor hydraulic machines were developed.

Materials and Methods. Materials included known methods of profile parameters calculation, based on application of classical formulas of hypocycloidal equidistant used for outlining profiles of teeth of working elements of gerotor machines. The basic research method was modeling the gerotor machine profile using Mathcad computer mathematics system. Calculated data were obtained for the selected ranges of varying parameters, processed through the univariate regression analysis.

Results. An algorithm for analyzing the tooth profile smoothness was developed. Two target parameters were defined: the cross-sectional area of the end profile, which affects the productivity; the smallest reduced radius of contact that determines operability of the working body. A technique for calculating the target parameters at the early stage of design was proposed. A number of optimal values of profile parameters according to the criteria of productivity and operability of gerotor machine was obtained. The dependences providing the optimum values of profile parameters at the stage of designing were constructed.

Discussion and Conclusion. The developed methodology makes it possible to obtain an assessment of the performance and operability of the gerotor hydraulic machine at the design stage of the working body. The research results can be used in mechanical engineering when designing gerotor hydraulic machines in order to improve their technical and operational characteristics.

Keywords: hydraulic machine, gerotor gearing, hypocycloid, rotor, stator, contact, open area

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Оптимизация геометрических характеристик циклоидальных профилей героторных гидромашин

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Аннотация

Введение. Производительность и надежность работы героторных гидромашин зависит от геометрических параметров профиля циклоидального зацепления. Существующие методики расчета и оптимизации параметров профиля громоздкие, многокритериальные, сложные для практического применения. Поэтому актуальной представляется проблема создания методики расчета параметров профиля рабочего органа героторной машины, пригодной для инженерных расчетов на этапе эскизного проектирования. В связи с этим целью данной работы является модернизация методики проектирования геометрии профилей торцевого сечения гипоциклоидальных зубчатых зацеплений, используемых в героторных гидравлических машинах, и анализ возможностей их оптимизации при предварительном проектировании. В ходе исследования была использована система компьютерной математики Mathcad, проведены численные эксперименты для изучения влияния геометрических параметров профилей на производительность и работоспособность героторной гидравлической машины. на основе полученных и проанализированных данных выработаны рекомендации по проектированию оптимальных профилей торцевого сечения героторных гидравлических машин.

Материалы и методы. Материалы включают в себя известные методики расчета параметров профиля, основанные на применении классических формул эквидистант гипоциклоид, используемых для очерчивания профилей зубьев рабочих органов героторных машин. Основным методом исследования — моделирование профиля героторной машины с помощью системы компьютерной математики Mathcad. Получены расчетные данные для выбранных диапазонов варьируемых параметров, обработанные методом регрессионного однофакторного анализа.

Результаты исследования. Разработан алгоритм анализа гладкости профилей зубьев. Определены два целевых параметра: площадь сечения торцевого профиля, влияющая на производительность, и наименьший приведенный радиус контакта, определяющий работоспособность рабочего органа. Предложена методика по расчету целевых параметров на ранней стадии проектирования. Получен ряд оптимальных значений параметров профиля по критериям производительности и работоспособности героторной машины. Построены зависимости, позволяющие установить оптимальные значения параметров профиля на стадии проектирования.

Обсуждение и заключение. Разработанная авторами методика дает возможность на этапе проектирования рабочего органа получить оценку производительности и работоспособности героторной гидромашин. Результаты исследований могут быть использованы в машиностроении при проектировании героторных гидромашин для улучшения их технико-эксплуатационных характеристик.

Ключевые слова: гидравлическая машина, героторное зацепление, гипоциклоида, ротор, статор, контакт, площадь живого сечения

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Introduction. Gerotor hydraulic machines are increasingly being used in the construction and operation of oil and gas wells. The reason is their structural and operational advantages, the primary of which are low delivery unevenness for pumps or torque for motors, compact overall dimensions.

In the scientific literature, a large place is occupied by studies devoted to the operation of gerotor hydraulic machines. The strong geometric effects of the rotor profile on the performance and reliability of screw gerotor hydraulic

machines were noted by D. and F. Baldenko [1]. The basic unit of such a machine is a multithread screw gerotor mechanism — a cylindrical planetary gear train of internal gearing, consisting of a stator and a rotor, with a difference in the number of teeth equal to one. The stator profile in the end section contacts the rotor profile, forming closed platforms. The authors also analyzed the technology of manufacturing gears.

R. Iyer, V. Jatti and others investigated the feasibility of mathematical models in the design of screw pumps [2]. They also presented the results of modeling various sizes of twin-screw pumps when pumping water, a mixture of water and air. Hong-Seok Park and Xuan-Phuong Dang proposed a CAD model optimization process using the results of CAE modeling and Mathcad calculations [3]. Other authors (A. Lebedev, S. Kireev, M. Korchagina, S. Vlaskin, A. Efimov) noted in their publications the effectiveness of designing various objects and processes based on parametrization and subsequent optimization of the design according to one or more target parameters [4–6].

In their work, F.D. Baldenko and Yang Yao have emphasized that the open area does not depend on the relative position of the rotor and stator profiles and determines the pump performance. They analyzed the effect of dimensionless geometric profile parameters on the open area [7].

Jie Chen, He Liu, Fengshan Wang, Guocheng Shi, Gang Cao and Hengan Wu developed a finite element model of the working body of a screw pump with a surface drive for the oil and gas industry [8]. Based on the results of modeling and experimental tests, they concluded that the gaps in the coupling and the thickness of the stator were the two key factors affecting the optimized target parameter — the volumetric efficiency of the pump.

New data were obtained by J. Gamboa, A. Olivet and S.H. Espin when modeling the working body of a multilobe screw pump [9]. These authors determined a significant relationship between the sliding clearance area and the pressure drop associated with the mechanical properties of the stator material.

Yang Yao and F. Baldenko investigated the effect of dimensionless coefficients of the end profile of a gerotor hydraulic machine on the characteristics of cycloid engagement, which provided optimization based on geometric and kinematic criteria [10]. I.A. Lyagov, F.D. Baldenk, A.V. Lyagov, V.U. Yamaliev and A.A. Lyagova proposed a methodology for selecting the optimal configuration of power sections when designing sectional downhole motors, noting the urgency of the problem of reducing the overall parameters of the screw pump length [11].

Yu.A. Korotaev, A.N. Alpatov, A.V. Sobolev and others have come to the conclusion that the most effective is mesh profiling from the initial contour of the rail outlined by the equidistant shortened cycloid [12]. With regard to the profiles obtained from the equidistant of an ordinary hypocycloid, the authors pointed out their major drawback: they were built as special and did not provide for tension in the engagement. Yu.A. Korotaev and D.A. Goldobin drew attention to the problem associated with the need to increase the performance of screw pumps, and the resulting unacceptable rigidity conditions under the production of stators and rotors [13].

A.F. Minikaev, V.A. Pronin, D.V. Zhignovskaya, Yu.L. Kuznetsov, F.D. Baldenko, A.E. Kovalenok developed a methodology for studying the stress-strain state of the stator during contact interaction with the rotor of a screw working body of a hydraulic machine using CAE modeling. They have stated that the classical Hertz theory of contact interaction cannot be applied to determine contact stresses in the rotor and stator due to a number of features: change in the curvature of the contacting surfaces of the rotor and stator; different thickness of the elastic lining along the circumference of the housing; presence of preload in a pair; complex planetary motion of the rotor, accompanied by a combination of friction rolling and sliding; displacement of the radial force vector relative to the normal at the point of contact.

Summing up the review of scientific publications on the design of a gerotor hydraulic machine, it should be noted that most researchers in this field strive to improve the efficiency of this machine by increasing the open area and reducing the overall dimensions of the working body. Optimal results were achieved by applying and varying the coefficients of off-centroidness, tooth shape, relative sliding speed, rail displacement, etc. Calculation methods were cumbersome, they were difficult to apply in practice. End profiles were built, as a rule, on the basis of shortened hypocycloids, which caused problems with providing kinematic accuracy of movement, contact interaction of the rotor and stator. Some authors (Yu.A. Korotaev, A.N. Alpatov, A.V. Sobolev, et al.) pointed out the impossibility of creating tension in the case of using an ordinary hypocycloid, but tension could be created through changing the equidistant distance of the rotor and stator profiles.

Materials and Methods. To obtain profiles of the rotor and stator in the end section of the working body of the gerotor hydraulic machine, we specify the following set of initial parameters (Fig. 1):

- 1) R_k — profile contour radius, which determines the diameter of circle C_1 , describing the profile of the stator (mm).

In fact, this parameter assigns the radial dimension of the gerotor machine, which is particularly important in the oil and gas industry when the projected object needs to be placed, e.g., in the trunk of the production well column;

- 2) z_1 — number of rotor teeth;

- 3) R_r — radius of the roller, which determines the distance of the equidistant line of the rotor and stator profiles relative to the corresponding hypocycloids (mm);
 4) Δ — tension in the coupling of the rotor and stator (mm).

Recall the principle of formation of hypocycloidal engagement profiles, which consists in the fact that inside circle C_2 (base circle of the stator hypocycloid), circle C_3 (base circle of the rotor hypocycloid) rolls without sliding. The center of circle C_3 is always at distance e (eccentricity) from the center of circle C_2 .

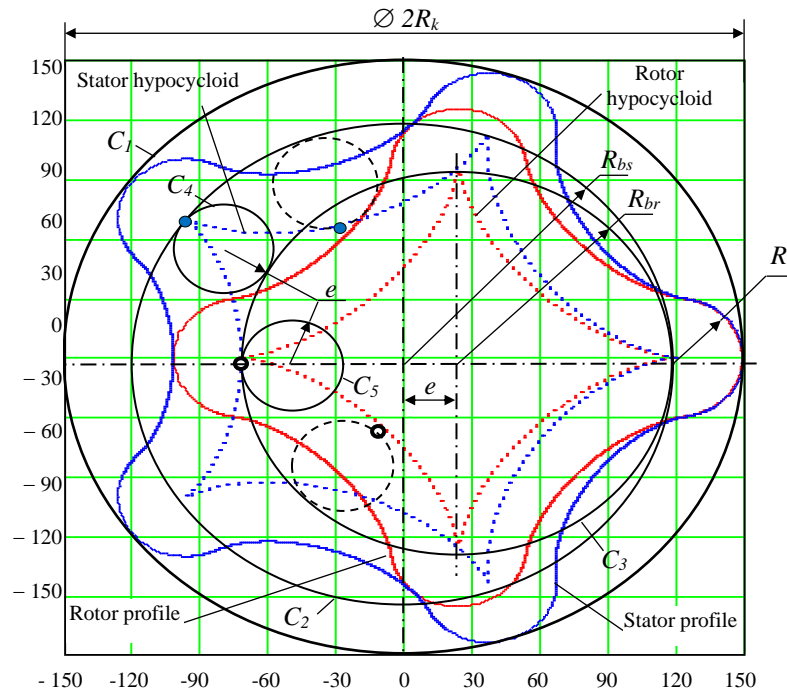


Fig. 1. Diagram of formation of the hypocycloidal profile of the end section of the working body of the gerotor machine,
 $z_1 = 4$, $R_k = 150$ mm, $R_r = 30$ mm

Inside circle C_3 , circle C_5 (generating circle of the rotor hypocycloid), one point of which forms the rotor hypocycloid, rolls without sliding. Inside circle C_2 , circle C_4 (generating circle of the stator hypocycloid), one point of which forms the stator hypocycloid, rolls without sliding. Note that the radii of circles C_5 and C_4 are equal to eccentricity e . The rotor and stator profiles are formed as equidistant lines deposited at distance R_r from the corresponding hypocycloids.

In this setting, the rotor and stator profiles provide an ideal contact under eccentric rotation of the rotor. When required to provide tension/clearance in the engagement, the equidistant distance R_r for the rotor profile should be changed by Δ .

Let us move on to the mathematical description of the method of forming the profiles of the end section of the rotor and stator. The number of stator teeth is equal to

$$z_2 = z_1 + 1. \quad (1)$$

The radius of the base circle of the stator hypocycloid R_{bs} is determined from the condition (Fig. 1):

$$R_{bs} = R_k - R_r. \quad (2)$$

We determine eccentricity e of the transmission from the condition that the length of circle C_2 is equal to integer z_2 of the lengths of generating circle C_4 with radius e :

$$2pR_{bs} = 2pe z_2,$$

then,

$$e = R_{bs} / z_2. \quad (3)$$

Note that for $z_1 = 1$ (Moineau mechanism) according to other formulas:

$$e = R_k / 3uR_r = 2e, \text{ if } z_1 = 1. \quad (4)$$

The radius of the base circle of the rotor hypocycloid R_{br} is equal to (Fig. 1):

$$R_{br} = R_{bs} - e. \quad (5)$$

We introduce a function to determine the radius of the base circle of the rotor or stator, fulfilling conditions (2) and (5):

$$R(z) = R_k - R_r - e \cdot (z_2 - z). \quad (6)$$

Parametric formulas of the hypocycloid of the rotor and stator in the Cartesian coordinate system XOY:

$$\begin{aligned} x(\varphi, z) &= R(z) \cdot \left[(1 - 1/z) \cos(\varphi/z) + (a/z) \cos((1 - 1/z)\varphi) \right] \\ y(\varphi, z) &= R(z) \cdot \left[(1 - 1/z) \sin(\varphi/z) - (a/z) \sin((1 - 1/z)\varphi) \right] \end{aligned} \quad (7)$$

where z — number of branches of the hypocycloid of rotor z_1 or stator z_2 , a — velocity factor of the hypocycloid. In the case of an ordinary hypocycloid in theory, $a = 1$. However, in practical calculations using computer mathematics systems (Mathcad, MATLAB) at $a = 1$, difficulties arise with calculating equidistant profiles of the rotor and stator. Therefore, in the model, we assume $a = 0.99$, which is quite justified to achieve our goals; φ — rotation angle of the radius vector drawn from the origin to each point of the hypocycloid. If the hypocycloid must be constructed from N_p points, then

$$\varphi(i, z) = 2 \cdot \pi \cdot i \cdot z / N_p, \quad i = 0 \dots N_p. \quad (8)$$

The profiles of the rotor and stator are determined from the well-known equidistant trajectory formulas relative to the hypocycloids of the rotor or stator at distance R_r taking into account the tension in the coupling Δ .

$$\begin{aligned} X_p(\varphi, z, \Delta) &= x(\varphi, z) + (R_r + \Delta) \cdot \frac{\partial y(\varphi, z)}{\partial \varphi} / \sqrt{\left(\frac{\partial y(\varphi, z)}{\partial \varphi} \right)^2 + \left(\frac{\partial x(\varphi, z)}{\partial \varphi} \right)^2} \\ Y_p(\varphi, z, \Delta) &= y(\varphi, z) - (R_r + \Delta) \cdot \frac{\partial x(\varphi, z)}{\partial \varphi} / \sqrt{\left(\frac{\partial y(\varphi, z)}{\partial \varphi} \right)^2 + \left(\frac{\partial x(\varphi, z)}{\partial \varphi} \right)^2} \end{aligned} \quad (9)$$

During transmission operation, the center of the rotor moves along a circle with radius e and rotates by angle $\alpha = 0 \dots 2\pi$. At the same time, all points of the rotor make an additional rotation by angle ψ , determined by the function

$$\psi(\alpha) = \alpha \cdot (1 - z_2 / z_1). \quad (10)$$

The rotor rotation model is described by parametric coordinate transformation functions when moving and rotating an object:

$$\begin{aligned} X_v(\alpha, \varphi, \Delta) &= e \cdot \cos(\alpha) + X_p(\varphi, z_1, \Delta) \cos(\psi(\alpha)) - Y_p(\varphi, z_1, \Delta) \sin(\psi(\alpha)) \\ Y_v(\alpha, \varphi, \Delta) &= e \cdot \sin(\alpha) \cdot \text{if}(z_1 = 1, 0, 1) + X_p(\varphi, z_1, \Delta) \sin(\psi(\alpha)) + Y_p(\varphi, z_1, \Delta) \cos(\psi(\alpha)) \end{aligned} \quad (11)$$

In expression (11), a conditional function is applied that zeroes the movement along Y axis in the case of rectilinear movement of the rotor in the stator cavity at $z_1 = 1$.

The experience of using this technique has shown that the equidistant profiles of the rotor and stator with an increase in the equidistant distance or radius of the roller R_r can lose smoothness. This occurs at the points of transition from the equidistant circle to the equidistant hypocycloid. Figure 2 shows the problem of loss of smoothness of the stator profile line in the vicinity of ± 1 mm from the transition point of equidistant lines.

In mathematics, the formula for the function smoothness criterion given in parametric form is known. For the stator profile, such a function has the form

$$F_s(p, z) = \frac{\partial Y_p(p, z, 0)}{\partial p} / \frac{\partial X_p(p, z, 0)}{\partial p}, \quad (12)$$

where $z = z_2$;

p — angle of the radius vector of the stator profile point.

It should be noted that function F_s behaves unstable in the entire range $\varphi(i, z_2)$, and it is quite difficult to find any features of its behavior at the transition point of equidistant lines of interest to us. Therefore, the behavior of function F_s was studied only in the vicinity of the transition point when the angular parameter changes

$$p = \pi / z_2 - m \dots \pi / z_2 + m, \quad (13)$$

where m — angle of deviation from the angle of the radius vector of the transition point.

$$m = 2 \cdot p \cdot u, \quad (14)$$

where u — part of the angular sector of the stator profile to study smoothness in the vicinity of the transition point. Value u (for $z_2 = 2 \dots 12$) is determined from the empirical formula

$$u = 0.12 - 0.01 \cdot (z_2 - 1). \quad (15)$$

Figure 2 shows that with an increase in the radius of the roller in the vicinity of the transition point of equidistant lines, the stator profile turns from smooth to self-intersecting. The behavior of function F_s also changes, and when the self-intersection effect of the profile appears, a loop occurs (Fig. 2 c).

This effect is taken as a basis for the stator profile smoothness control algorithm proposed by the authors of the article (Fig. 3). It should be noted that, judging by numerous measurements, if the stator smoothness criterion is met, then the smoothness of the rotor profile does not need to be checked, since it is built on a hypocycloid with a smaller value of the radius of the base circle.

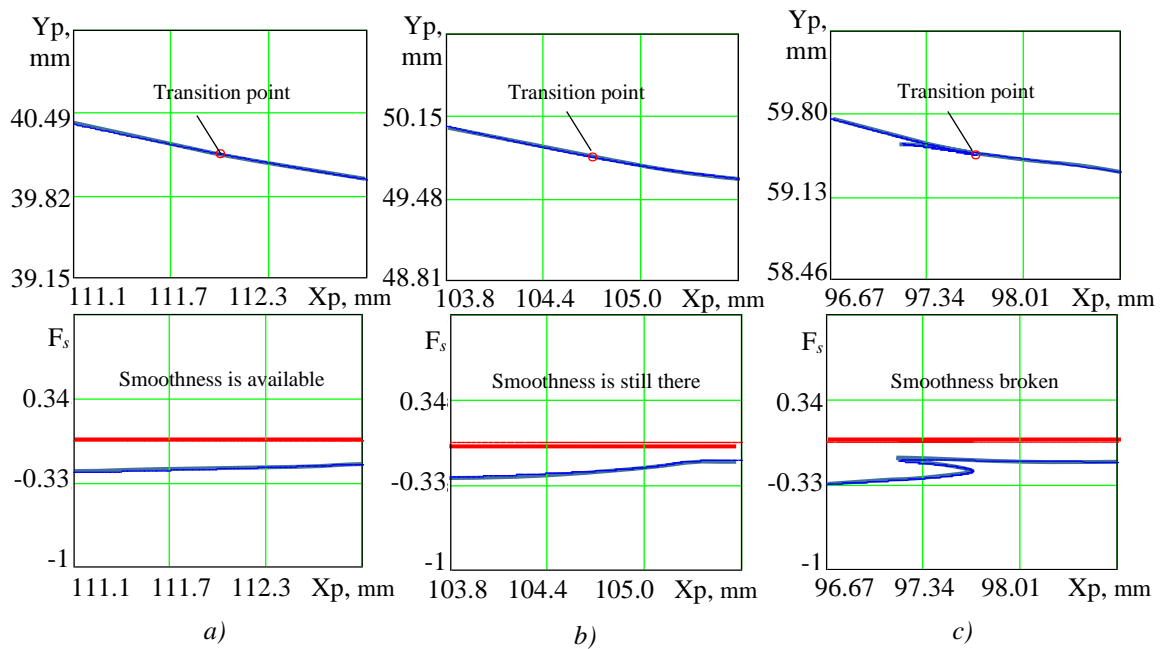


Fig. 2. Stator profile in the vicinity of ± 1 mm from the transition point of equidistant lines (upper graph) and values of function F_s (lower graph) at $z_1 = 3$ and $R_k = 150$ mm, for R_r : a — 40 mm; b — 50 mm; c — 60 mm

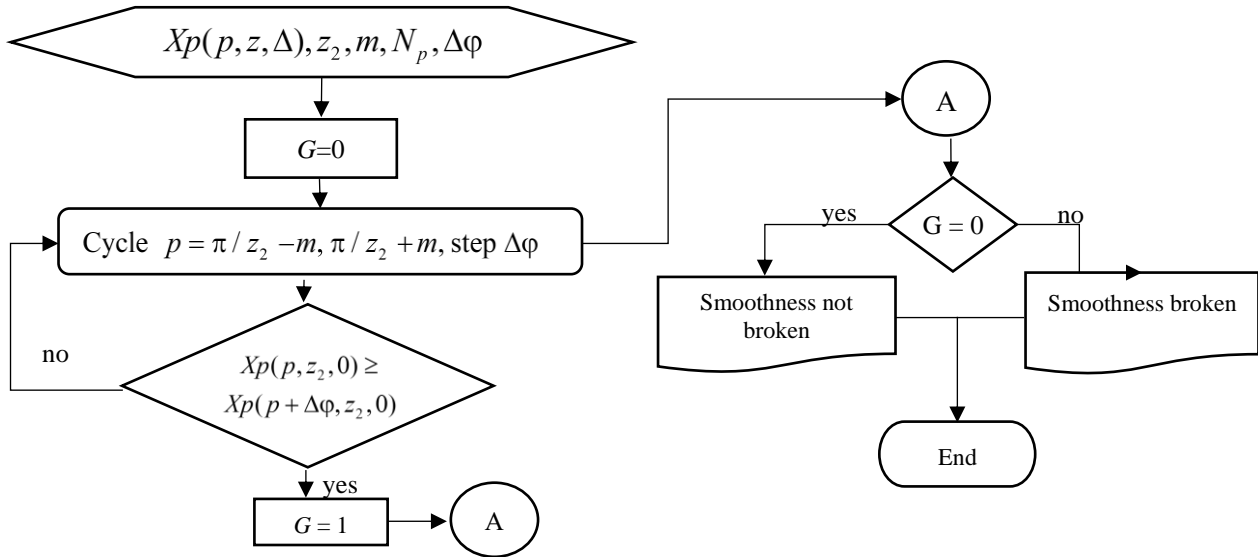


Fig. 3. Algorithm scheme for monitoring the stator profile smoothness

Control of the stator profile smoothness should always be carried out when designing the geometry of the end section of the gerotor machine. In order to study the parameter of the maximum radius of roller $R_{r\max}$, below which the smoothness of the curve was not violated, the authors conducted a series of numerical experiments where various values of the contour radius R_k and the numbers of rotor teeth z_1 were specified.

The first target parameter of this study is the open area of the working body S . This parameter is included in the formula for feeding the gerotor machine, and it determines its performance. With the available parametric functions describing the profiles of the rotor and stator, the task of determining S is reduced to calculating the difference in the areas of the parametric curves of stator S_s and rotor S_r (9). The formula for the area of the parametric curve is known from mathematics; therefore, in the authors' methodology, it looks like this:

$$S = S_s - S_r = \int_0^{z_2 \cdot 2\pi} \left(X_p(p, z_2, 0) \cdot \frac{\partial Y_p(p, z_2, 0)}{\partial p} \right) dp - \int_0^{z_1 \cdot 2\pi} \left(X_p(p, z_1, \Delta) \cdot \frac{\partial Y_p(p, z_1, \Delta)}{\partial p} \right) dp \quad (16)$$

The second target parameter of the study is the reduced radius ρ_{np} of curvature at the contact interaction of the rotor and the stator. Recall that according to Hertz theory, when two cylindrical surfaces come into contact, the reduced radius is calculated, which is in the denominator in the contact stress formula. Consequently, the magnitude of the contact stresses, along with the physical and mechanical parameters of the materials of the contacting surfaces, has the largest value at the lowest value of the reduced radius ρ_{np} , determined from the formula:

$$\rho_{np} = (\rho_r \cdot \rho_s) / (\rho_r + \rho_s). \quad (17)$$

where ρ_r — radius of the equidistant circle of the rotor, equal to parameter R_r ; ρ_s — radius of the stator profile curve.

Note that if $\rho_s > 1$, then the numerator in formula (17) always grows faster than the denominator; therefore, the smallest value $\rho_{np\min}$ is completely determined by the parameter of the smallest radius of the stator $\rho_{ct\min}$.

In the end section of the working body of the gerotor machine, at any rotation angle of the rotor, continuous contact of its equidistant circle and a complex curve of the stator profile is made. Based on the results of observations of the contact interaction dynamics, it can be concluded that the greatest stresses occur at the transition points of the equidistant hypocycloids in the equidistant circumference of the stator profile. Consequently, the smallest radius of the stator profile line $\rho_{ct\min}$ is also located in the transition zone of equidistant lines and determined by the formula known from mathematics:

$$\rho_{cm\min}(p) = \frac{\left[\left(\frac{\partial X_p(p, z_2, 0)}{\partial p} \right)^2 + \left(\frac{\partial Y_p(p, z_2, 0)}{\partial p} \right)^2 \right]^{\frac{3}{2}}}{-\left(\frac{\partial X_p(p, z_2, 0)}{\partial p} \right) \cdot \left(\frac{\partial^2 Y_p(p, z_2, 0)}{\partial p^2} \right) + \left(\frac{\partial^2 X_p(p, z_2, 0)}{\partial p^2} \right) \cdot \left(\frac{\partial Y_p(p, z_2, 0)}{\partial p} \right)}. \quad (18)$$

where $\rho = \pi / z_2$ angular parameter of the stator profile point at the junction of equidistant lines. In case of loss of smoothness of the stator line, parameter $\rho_{cr \min}$ tends to zero.

The technique described above was implemented in the Mathcad system of mathematical calculations. When performing calculations, the hypocycloid velocity factor $a=0.99$, $N_p = 40,000$.

At the first stage of research, it was planned to study the maximum radius of roller $R_{r \max}$, at which the stator profile line smoothness criterion G was zero. Variable parameters — contour radius R_k (100, 150, 200 mm) and the number of rotor teeth z_1 (from 1 to 7). Calculations were performed with a sequential increase in parameter R_r from 5 mm until the smoothness criterion G was triggered. The investigated parameter $R_{r \max}$ was fixed with an accuracy of 1 mm.

At the second stage, a series of calculations were carried out to study the effect of the roller radius R_r (from 5 mm to $R_{r \max}$), the number of rotor teeth z_1 (from 2 to 7) and the contour radius R_k (100, 150, 200 mm) on the open area S and the minimum reduced contact radius $\rho_{np \min}$.

The described method of calculating the profile geometry of the end section of the working body of the gerotor machine can be implemented in any other programming system or in a computer mathematics system.

Research Results. The data of numerical experiments to determine the parameter of the maximum radius of roller $R_{r \max}$, at which the stator profile line smoothness is preserved, are shown in Table 1.

Table 1

Maximum radius of roller $R_{r \max}$ (mm) at different contour radii R_k (mm)

z_1	$R_k = 100$	$R_k = 150$	$R_k = 200$
1	66.6	100	133
2	45	68	91
3	27	41	55
4	20	31	40
5	15	23	31
6	12	19	25
7	10	16	21

Based on the data in Table 1, graphs were constructed (Fig. 4 a). It can be seen that the estimated parameter $R_{r \max}$ grows proportionally with increasing contour radius R_k . We introduce into consideration the relative parameter $R_o = R_{r \max} / R_k$, whose graph is presented in Figure 4 b.

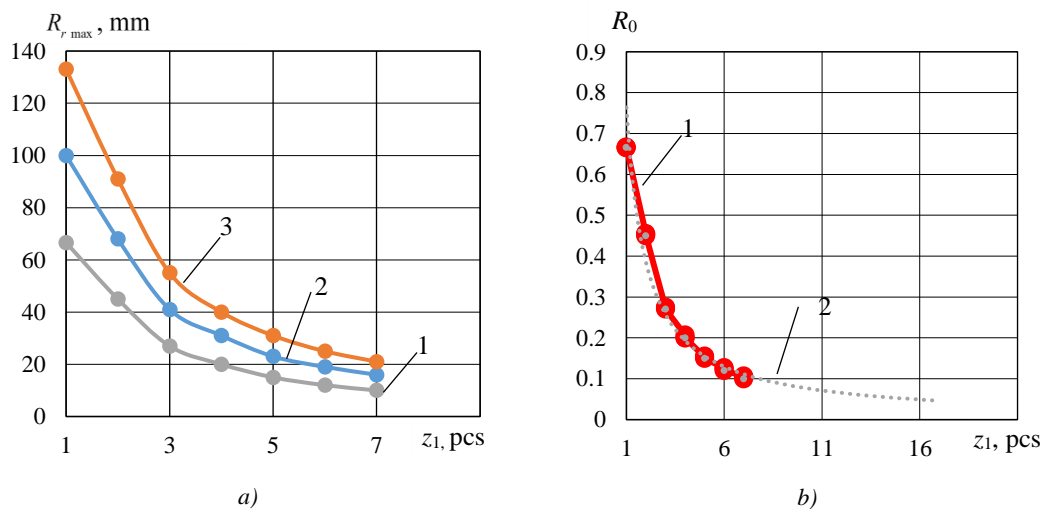


Fig. 4. Dependence of maximum radius of the roller on the number of teeth of the rotor:

a — $R_{r \max}$ and R_k : 1 — 100 mm; 2 — 150 mm; 3 — 200 mm; b — R_o (1) and trend line (2) with extrapolation to $z_1 = 16$

The empirical formula from which it is possible to determine the maximum allowable radius of roller $R_{r \max}$, has the form:

$$R_o = 0.768 z_1^{-0.996} \quad (19)$$

At $z_1 = 15$, $R_k = 150$ mm, $R_o = 0.0517$ mm, $R_{r \max} = 7.76$ mm, the profile smoothness is achieved at $R_r = 7.5$ mm.

At $z_1 = 10$, $R_k = 200$ mm, $R_o = 0.0775$ mm, $R_{r \max} = 15.5$ mm, the profile smoothness is achieved at $R_r = 15.5$ mm.

Thus, we can conclude about the satisfactory accuracy of empirical formula (19) for predicting the maximum allowable parameter $R_{r \max}$.

It should be noted that formula (19) was obtained at the values of the velocity factor $a = 0.99$. A decrease in a causes the smoothness of the profile with a more significant increase in parameter R_r . However, the shape of the cycloidal engagement tooth changes, and the conditions of contact interaction between the rotor and the stator are violated, specifically, in those places where the rotor should touch the stator with the sides of the teeth. Calculations show that at $a = 0.99$ and $R_r = 38$ mm, the smoothness criterion is at the limit of feasibility. Though at $a = 0.90$ and $R_r = 38$ mm, good smoothness of the stator profile is achieved, but the kinematic contact of the parts of the working body of the machine is disrupted, which can be restored only by a special selection of the values of the hypocycloid parameters of the rotor and stator. Therefore, the study of the influence of the velocity factor on parameter $R_{r \max}$ is beyond the scope of this research.

Table 2 presents the data of calculations of the target parameters of the studies for the number of rotor teeth $z_1 = 3$ at different values of contour radii. The obtained values of the open area S and the minimum reduced contact radius $\rho_{np \min}$ in the last two columns of Table 2 are given in a dimensionless form for ease of analysis.

Table 2

Calculation data of target parameters at $z_1=3$

R_r , mm	$\rho_{np \min}$, mm	S , mm ²	$S/\max(S)$	$\rho_{np \min} / \max(\rho_{np \min})$
$R_k = 150$ mm				
5	4.72	18,038	0.9712	0.2656
10	8.83	18,256	0.983	0.4969
15	12.28	18,415	0.9915	0.6911
20	14.98	18,513	0.9968	0.843
25	16.85	18,572	1	0.9482
30	17.77	18,530	0.9977	1
35	17.63	18,450	0.9934	0.9921
40	16.28	18,307	0.9857	0.9162
$R_k = 200$ mm				
10	9.14	32,275	0.9786	0.3857
20	16.37	32,738	0.9926	0.6909
30	21.36	32,982	1	0.9015
35	22.9	32,980	0.9999	0.9664
40	23.69	32,943	0.9988	0.9998
45	23.695	32,844	0.9958	1
50	22.82	32,685	0.991	0.9631
55	20.98	32,466	0.9844	0.8854
$R_k = 100$ mm				
5	4.57	8,068	0.9773	0.3857
10	8.18	8,184	0.9914	0.6903

R_r , mm	$\rho_{np \min}$, mm	S , mm ²	$S/\max(S)$	$\rho_{np \min} / \max(\rho_{np \min})$
$R_k = 150$ mm				
13	9.83	8,225	0.9964	0.8295
15	10.68	8,245	0.9988	0.9013
17	11.32	8,255	1	0.9553
20	11.85	8,235	0.9976	1
24	11.64	8,189	0.992	0.9823

Figure 5 shows graphs of variation in the target parameters of studies at $z_1=3$. It is easily seen on them that with an increase in the radius of roller R_r , there is a slight change in the open area S and a sufficiently strong change in the reduced contact radius, whose maximum value falls on a fairly narrow range R_r . This is of the greatest interest to the designer, since in this case, the contact stresses are minimal. Calculations for other values z_1 (from 2 to 7) showed approximately the same qualitative picture as in Figure 5.

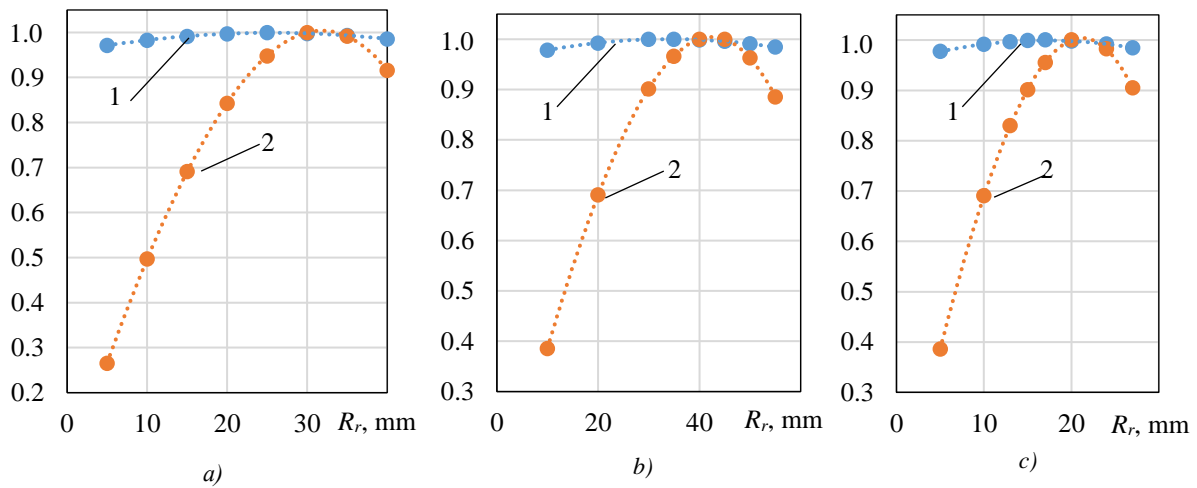


Fig. 5. Dependences of the target parameters of the study on the radius of roller R_r at $z_1 = 3$. a = 0.99. $\Delta = 0$: 1 — $S/\max(S)$; 2 — $\rho_{np \min} / \max(\rho_{np \min})$ (Table 2); R_k , mm: a — 150; b — 200; c — 100 mm

Table 3 shows the calculation data of the optimal radii of rollers $R_{r \text{ opt}}$ for various parameters z_1 and R_k . As a result of the analysis, it turned out that parameters $R_{r \text{ opt}}$ and R_k for one z_1 change almost the same. Therefore, if you enter the parameter of the relative optimal radius of the roller

$$R_{ro} = R_{r \text{ opt}} / R_k, \quad (20)$$

then, with the help of a single-factor dependence of $R_{ro} = f(z_1)$ -type, it is possible to determine the optimal radius of roller R_r at the preliminary stage of designing the working body of the gerotor machine.

As a result of similar reasoning, for parameters S and $\rho_{np \min}$, given in Table 3, dimensionless parameters of the relative reduced radius of contact $\rho_{np o}$ and relative open area $\rho_{np \min}$ were also introduced (Table 3):

$$S_o = S / R_k^2; \rho_{np o} = \rho_{np \min} / R_k \quad (21)$$

New relative values are also presented in Table 3.

Table 3

Optimal radii $R_{r\ opt}$ and relative engagement parameters

R_k , mm	z_1	$R_{r\ opt}$, mm	S , mm ²	$\rho_{np\ min}$, mm	R_{ro}	$\rho_{np\ o}$	S_o
200	2	68	36,103	43.2	0.340	0.216	0.903
150	2	51	20,318	32.2	0.340	0.215	0.903
100	2	34	9,015	21.65	0.340	0.217	0.902
100	3	21	8,235	11.85	0.210	0.119	0.824
150	3	32	18,530	17.8	0.213	0.119	0.824
200	3	42	32,800	23.1	0.210	0.116	0.820
200	4	28	30,015	15.32	0.140	0.077	0.750
150	4	21	16,880	11.5	0.140	0.077	0.750
100	4	14	7,495	7.63	0.140	0.076	0.750
100	5	10	6,860	5.35	0.100	0.054	0.686
150	5	15	15,441	8.1	0.100	0.054	0.686
200	5	20	27,452	10.81	0.100	0.054	0.686
150	7	9	13,050	4.7	0.060	0.031	0.580
200	7	12	22,100	6.2	0.060	0.031	0.582
100	7	6	5,780	3.2	0.060	0.032	0.578

It should be noted that all newly introduced relative values with a high degree of confidence obey approximating formulas that include one factor — the number of rotor teeth z_1 .

The formula for the relative optimal radius of roller R_{ro} has the form

$$R_{ro} = 0.9331 \cdot z_1^{-1.392} \quad (R^2 = 0.9965), \quad (22)$$

where R^2 — value of the approximation accuracy.

The formula for the relative reduced contact radius $\rho_{np\ o}$ has the form

$$\rho_{np\ o} = 0.635 \cdot z_1^{-1.534} \quad (R^2 = 0.9995). \quad (23)$$

The formula of the relative open area S_o has the form

$$S_o = 1.0175 - 0.0641 z_1 \quad (R^2 = 0.9911). \quad (24)$$

Let us consider an example of the application of the results obtained in the design of the geometry of the end section profile of the gerotor machine. We specify the following set of initial data: contour radius $R_k = 120$ mm, number of rotor teeth $z_1 = 4$, tension in the coupling $\Delta = 0$ mm (empirical formulas are obtained at zero tension). The relative optimal radius of roller R_{ro} according to formula (22):

$$R_{ro} = 0.9331 \cdot 4^{-1.392} = 0.135.$$

Absolute value of the roller radius $R_r = R_{ro} \cdot R_k = 0.135 \cdot 120 = 16.2$ mm.

Relative reduced contact radius $\rho_{np\ o}$ from formula (23):

$$\rho_{np\ o} = 0.635 \cdot 4^{-1.534} = 0.076.$$

Absolute value of the minimum reduced contact radius

$$\rho_{np\ min} = r_{np\ o} R_k = 0.076 \cdot 120 = 9.12 \text{ mm.}$$

Relative open area S_o from formula (24):

$$S_o = 1.0175 - 0.0641 \cdot 4 = 0.761.$$

Absolute value of the open area

$$S = S_o \cdot R_k^2 = 0.761 \cdot 120^2 = 10,958 \text{ mm}^2.$$

Mathcad calculations give the following values: $S = 10,820 \text{ mm}^2$, $\rho_{np \min} = 9.167 \text{ mm}$. As can be seen in this example, empirical formulas accurately predict the basic parameters of the geometry of the end section of the gerotor machine.

Discussion and Conclusion. The conducted research was aimed at solving the problem of improving the working bodies of gerotor hydraulic machines. It sufficiently corresponds to the works of domestic and foreign authors [10–15]. The key difference is that when constructing the rotor and stator profiles, ideal hypocycloids are used, providing the greatest kinematic accuracy of movement and contact interaction of the engagement elements. It was proposed to solve the well-known problem of providing tension in the coupling with respect to the difference in the parameters of the roller radius. A special algorithm was developed to control the smoothness of the profiles [12]. The optimal geometry of the profile, as in [10], was proposed to be determined on the basis of the maximum open area of the working body of the gerotor machine. But at the same time, the reduced contact radius should also have the largest value, as a guarantee of the lowest contact stresses. In this study, based on a large number of calculations using a purpose-built Mathcad program, in the range of rotor teeth z_1 from 2 to 7 and contour diameter R_k from 100 to 200 mm, optimal values of the equidistant hypocycloid distance, or roller radii R_r , at which the best combination of the reduced contact radius and the open area of the working body of the gerotor was achieved. It turned out that the introduction of relative values of the radius of the roller, the open area, and the reduced contact radius provided obtaining absolute values of the same magnitudes for any contour radius within the established ranges using simple empirical dependences at an early stage of design.

The developed technique makes it possible to evaluate the performance and operability of the gerotor machine at the design stage of the rotor and stator profiles. In the future, similar studies can be carried out for epicycloid and mixed engagements in gerotor hydraulic machines.

The research results can be used in mechanical engineering under the design of gerotor hydraulic machines to improve their operating characteristics.

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